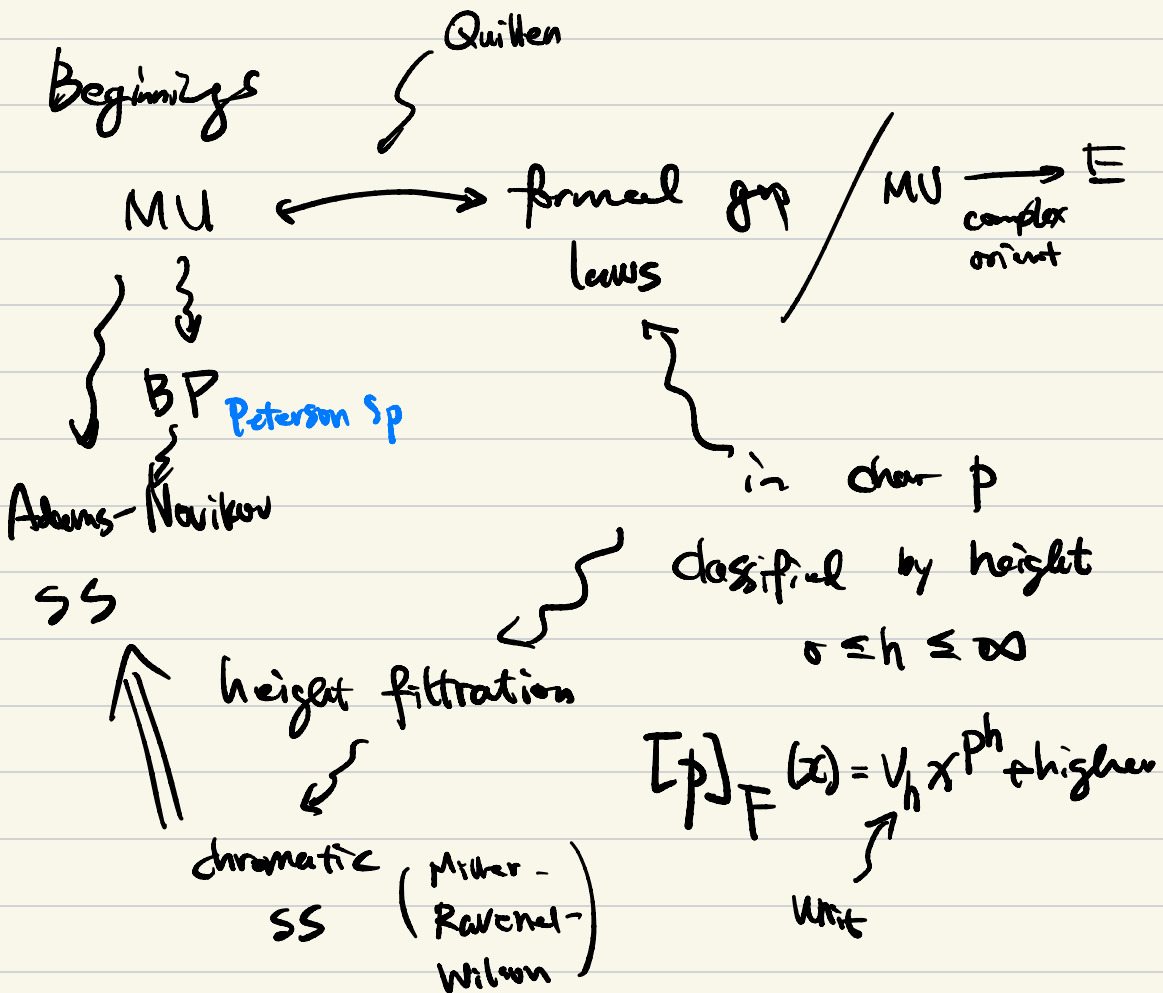




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## Introduction to Chromatic Homotopy Theory



Has homotopical origins

$K(n)$  Morava  $E$ -theory

$$K(n)_* \cong \mathbb{F}_p [v_n^{\pm 1}] \quad n \geq 1$$

with a fgl (Honda)

$$[p]_{\mathbb{F}_p}(x) = v_n x^{p^n}$$

eg  $v_0 = p$ ,  $K(0) = H\mathbb{Q}$

$$K(1) = L/p$$

↑  
Adams  $L$ -summand of  $KU$

$S_p \rightsquigarrow$  localize  $L_{K(n)}$

$$L_n = L_{K(n)} \vee \dots \vee K(n)$$

• Chromatic convergence (Hopkins - Ravenel)

$$S^0(p) \cong \lim_n L_n S^0$$

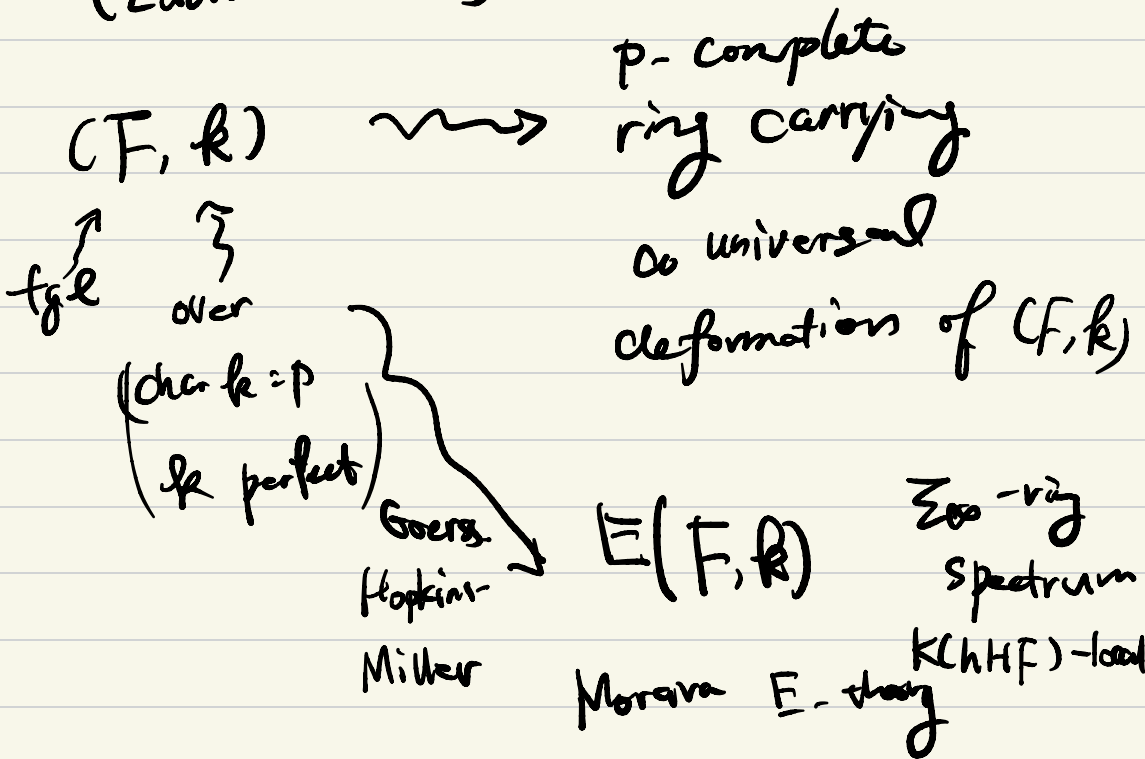
• Fracture squares

$$\begin{array}{ccc} L_n & \longrightarrow & L_{K(n)} \\ \downarrow & \lrcorner & \downarrow \\ L_{n-1} & \xrightarrow{\quad \leftarrow \quad} & L_{n-1} L_{K(n)} \end{array} \quad \left( \leftarrow \text{chromatic splitting conjecture} \right)$$

$\rightsquigarrow K(n)$ -local homotopy theory

Formal grp laws have nice deformations

(Lubin - Tate)



functoriality  $\rightsquigarrow$   $\text{Aut}(F, k)$  acts on  $E(F, k)$   
via  $\Sigma_{\infty}$ -ring maps

$\nearrow$   
nice profinite grp

Notation  $E_n = E(F_n, \mathbb{F}_p^n)$  ;  $G_n = \text{Aut}(F_n, \mathbb{F}_p^n)$   
Marava stabilizer grp

eg.  $E_1 = K\hat{U}_p$  ,  $G_1 = \mathbb{Z}_p^\times$  acts via  
 Adams operations

eg.  $E_2$  related to top. mod. forms

In general,  $\pi_* E_n \cong (\pi_0 E) [u^{\pm 1}] \quad |u| = -2$

↑ Lubin-Tate ring

$$\pi_0 E \cong \mathbb{Z}_p[[u_1, \dots, u_{n-1}]]$$

Thm <sup>Marava</sup> (Deviratz Hopkins)  $S_{K(n)}^0 \cong E_n^{hG_n}$

Rognes:  $S_{K(n)}^0 \rightarrow E_n$  is a profinite  
Galois extension  
(w/ Gal grp  $G_n$ )

Baker-Richter:  $E_n$  is (almost) the  
dyg. closure of  $S_{K(n)}^0$   
(need  $E(F_n, \bar{F}_p)$ )

$E(F, \bar{k})$  is alg. closed in sense  
of chromatic Nullstellensatz  
(Burklund - Schlenk - Yuan)

In fact  $SP_{K(G)} \cong (Mod \quad (E_n))^{hG_n}$

$x \mapsto (E_n \wedge x)$   
 $\xrightarrow{hG_n}$

$K(G)$  - local  
 $E_n$ -modules  
 Mathew, Mor, ...

descent

How to understand  $( )^{hG_n}$  ?

$n=1$   $G_1 = \mathbb{Z}_p^x$

$p$ -odd:  $S_{K(G)} = \mathbb{F}_1^{h\mathbb{Z}_p^x}$

$\rightsquigarrow S_{K(G)} \rightarrow KU_p^\wedge \xrightarrow{\psi^l} KU_p^\wedge$

( Adams - Baird - Ravenel Bousfield )



$$G_2 \cong \mathbb{Z}^3 \subset \mathbb{Z}_2^X,$$

$$p=2$$

$$S_{K(1)}^0 \xrightarrow{h_{G_2}} E_1^{h_{G_2}}$$

$n=2$  Goerss-Henn-Makowald-Rezk: resolution

$$(p=2) \quad S_{K(2)}^0 \rightarrow E_2^{hF} \rightarrow \dots$$

length  $4=2^2$

vertical  
cohomology  
dim

finite subgroups of  $G_2$

$$G_n \quad \text{vcd} = n^2 \quad \text{length } n^2$$

finite resolution in terms of

the finite subgroups  $F \subset G_n$

(i.e.  $S_{K(n)}^0$  res in terms of  $E_n^{hF}$ )

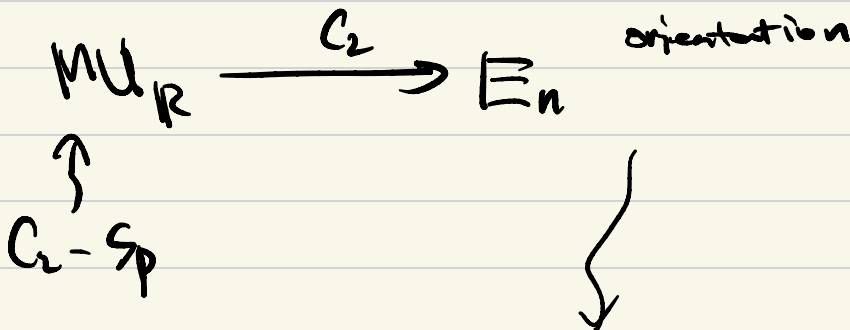
→ Study  $E_n^{hF}$

Finite  $p$ -subgroups are  $C_{p^i}$  or

$(p=2) Q_8$

$p=2$   $\{ \pm 1 \} = C_2 \subset G_n$

(Hahn - Shi):



$E_n$  Bond  $\rightarrow$  genuine  $F$ -space

$\Sigma_{\infty}$ -ring  $\rightarrow$  "genuine  $E_{\infty}$ "  
 $Sp$   $\rightarrow$   $B\text{Lambert}$  or  $N_{\infty}$   
still

$\rightsquigarrow C_2 \in F$

$N_{C_2}^F MU_R \xrightarrow{F} F$

$\nearrow$   
HHR norm